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RELAXATION METHOD OF SOLVING ELECTRICAL NETWORK PROBLEM OF "TWO WIRE D. C. TRANSMISSION LINES"

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ABSTRACT The present paper shows the application of relaxation method in the solution of D C Two Wire Transmission of Electrical Power with an illustration The principle of relaxational solution of D C networks having usual circuit conditions is utilised The results obtained thereby are compared with those calculated by the conventional method of network analysis It also points out the utility of relaxation method in cases where a number of informations are wanted simultaneously

INTRODUCTION

The calculations of D C Two Wire Transmission lines when conditions at the sending and receiving ends are given, are quite familiar There are different methods for such calculations and the problems can be solved according to the supplied informations of the sending and receiving end conditions

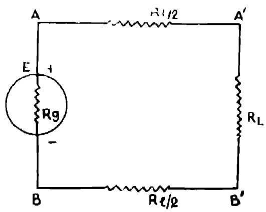

Fig. 1

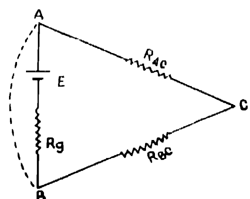

Fig. 2

When the receiving end terminals of a D.C Two Wire Transmission line network are terminated by load resistance as shown in Fig 1 the same network can be transformed into a convenient form as shown in Fig 2. This form of network is solved by relaxation method and it is shown that such a relaxational solution gives a number of useful informations simultaneously.

Relaxation method was applied by Southwell and Black (1938). to solve the problem of D C. networks and the utility of the application of relaxation technique for solving the network shown in Fig. 2.. is clearly shown here.

PRINCIPLE OF THE METHOD

In this method an electrical theorem in regard to the heating effects of steady currents has been used (Southwell and Black, 1938), which is enunciated below:

"In a network of conductors to which specified currents are supplied at two or more nodal points, the actual distribution of currents is such that the total generation of heat less twice the energy expended in supplying the specified currents from a source at datum potential has its minimum value consistent with the satisfaction of Kirchhoff's second law."

Let two nodal points A and C, joined by a conductor of resistance R_{AC}, be considered as shown in the network diagram represented by Fig 2. If V_A and V_C be the potential at A and C, by Ohm's law a current $\frac{V_A - V_C}{R_{AC}}$ will flow from A to C. Denoting the currents flowing towards A and C by I_A and I_C respectively, it can be written as

$$I_A = -I_C = g_{AC}(V_A - V_C), \text{ where } g_{AC} = 1/R_{AC}$$

Considering all the condctors connected to A, it is obtained as,

$$\sum_A g_{AC}(V_C - V_A) + I_{A0} = 0 \qquad (1)$$

where I_{A0} stands for the current supplied to A from outside. The heat generated in AC is measured by $g_{AC}(V_A - V_C)^2$ and so the total generation of heat in the network is given by,

$$2H = \sum_m g_{AC}(V_A - V_C)^2 \qquad \ldots \quad (2)$$

where $\sum_m$ denotes a summation extending to every conductor. Also if the current is supplied to A from an outside source at datum potential V_0 the rate of expenditure of energy is measured by $I_{A0}(V_A - V_0)$ and the total expenditure of energy is measured by,

$$\sum_n\{I_{A0}(V_A - V_0)\} = -E \qquad \ldots \quad (3)$$

where $\sum_n$ denotes a summation extending to every nodal point. So equation (1) is typical of the conditions for a minimum value of the quantity,

$$Q = H + E = \tfrac{1}{2} \sum_m \{g_{AC}(V_A - V_C)^2\} + \sum_n\{I_{A0}(V_A - V_0)\} \qquad \ldots \quad (4)$$

as it is equivalent to,

$$\frac{\delta Q}{\delta V_A} = \frac{\delta}{\delta V_A}(H + E) = 0$$

As the source of E M F., i.e. generator is involved it may be assumed that the whole E M F., of the source is utilized to pass current to earth through its own resistance and the datum distribution for the known current to enter and to leave the network at nodal points is obtained Afterwards it is simply required to calculate and superpose the effects of neutralising currents at those points. With this modification the problem can be solved conveniently by relaxation method taking the help of the above theorem.

If A and B be joined by a wire of zero resistance, shown by dotted line, the current passing through the source E from A to B would return by that wire and hence in the datum distribution a current of E/R_G enters the system at A and leaves at B where R_G is the internal resistance of the generator Now it is required to calculate and superpose the current distribution which when the neutralising currents $+E/R_G$ and $-E/R_G$ are supplied at B and A to the network after the source of E M F is removed.

Using equation (4), the expression for Q and the residuals can be written as follows :

$$2Q = \frac{(V_C - V_A)^2}{R_{AC}} + \frac{(V_C - V_B)^2}{R_{BC}} + \frac{(V_A - V_B)^2}{R_G} + 2\frac{E}{R_G}\{V_0 - V_B - (V_0 - V_A)\} \quad \ldots \quad (5)$$

where $$R_{AC} = \frac{R_l + R_L}{2} = R_{BC}$$

Hence,

$$\frac{\delta Q}{\partial V_C} = -\frac{V_C - V_A}{R_{AC}} - \frac{V_C - V_B}{R_{BC}} - 0 = F_C \text{ initially}$$

$$\left.\begin{aligned} -\frac{\partial Q}{\partial V_A} &= \frac{V_C - V_A}{R_{AC}} - \frac{V_A - V_B}{R_G} - \frac{E}{R_G} = -\frac{E}{R_G} = F_A \text{ initially} \\ -\frac{\partial Q}{\delta V_B} &= \frac{V_C - V_B}{R_{BC}} + \frac{V_A - V_B}{R_G} + \frac{E}{R_G} = +\frac{E}{R_G} = F_B \text{ initially} \end{aligned}\right\} \quad (6)$$

Now on liquidating the residuals obtained initially the required values of voltages at A, B and C can be found out The method is illustrated by the following example.

ILLUSTRATION

For illustration the following example worked by Corcoran (1949), using a different method is considered.

In the arrangement of the Two Wire D C. Transmission of Electrical Power shown in Fig. 1, the generated voltage $E = 110$ volts, internal resistance of the

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generator $R_G = 0.1$ ohm and the line resistance $R_l = 0.9$ ohm and the load resistance R_L is so adjusted that $V_{A'B'} = 100$ volts. The efficiency of transmission and the voltage regulation of the transmission line are to be found out.

The value of the load resistance R_L can be calculated to be 10 ohms from the supplied data. Then using the relations (5) and (6) the basic operation table (Table I) can be easily obtained (Allen 1954).

In the basic operations suitable positive increments are given to the unknowns and the corresponding changes in the residuals are found out (Table I, operation number 1 to 3). The block operation (Table II, operation number 2) in which equal increments are added to more than one unknown simultaneously to bring about the changes in the residuals quickly is carried on with the help of the basic operations. In preparing the relaxation table (Table II) both the basic operation and the block operation are used and the residuals are liquidated in only two steps (Table II operation number 1 and 2).

TABLE I

Basic Operation Table

Operation Number	δV_C	δV_A	δV_B	δF_C	δF_A	δF_B
1	5.45			2	53.5	1
2		5.45		1	55.5	54.5
3			5.45	1	54.5	−55.5

TABLE II

Relaxation Table

Operation Number	δV_C	δV_A	δV_B	F_C	F_A	F_B
	V_C	V_A	V_B 0	0	−1100	1100
1	0	−100	0	20	10	10
2	0	54.5	54.5	0	0	0
	0	54.5	54.5	0	0	0

TABLE III

Comparison of values

	Values Calculated by Relaxation Method	Values Calculated by Conventional Method of Network Analysis
Efficiency Transmiss	0.918	0.918
Voltage Regulatio	0.09	0.09

Hence the potential of B with respect to A is 109 volts and that of C relative to A is 54.5 volts. Therefore $V_{AB} = 109$ volts so that the efficiency of transmission is 0.918 and the voltage regulation of transmission line is is 0.09. These values are exactly the same as those obtained by the conventional method of network analysis shown by Table III.

DISCUSSION

The method described in this paper is a useful one as it gives out a number of informations at a time viz. potentials at A, B and C simultaneously. Also with some practice of relaxation method the table can be obtained with little difficulty and hence little time is required for the whole operation compared to other methods. But this method requires the value of the internal resistance of the source of *E.M.F.* or the resistance of the path AB to calculate the current in the datum distribution.

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